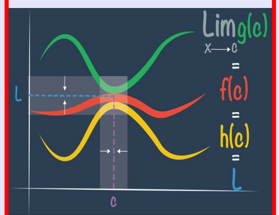
Math 261

Spring 2023

Lecture 22



Feb 19-8:47 AM

Class QZ 5

Given $f(x) = x^5 \cdot \tan x$

1) Find $f'(x) = \frac{d}{dx}[x^5] \cdot \tan x + x^5 \cdot \frac{d}{dx}[\tan x]$

$= \boxed{5x^4 \cdot \tan x + x^5 \cdot \sec^2 x}$ ✓

2) Find $f'(0) = 5 \cdot 0^4 \cdot \underbrace{\tan 0}_{0} + 0^5 \cdot \underbrace{\sec^2 0}_{1} = \boxed{0}$ ✓

Mar 14-9:43 AM

$f(x) = \sqrt{x^3+1}$

Find $f'(x)$

$f(x) = (x^3+1)^{1/2}$

$f'(x) = \frac{1}{2}(x^3+1)^{\frac{1}{2}-1} \cdot 3x^2$

$f'(x) = \frac{3x^2}{2(x^3+1)^{1/2}}$ $\quad f'(x) = \frac{3x^2}{2\sqrt{x^3+1}}$

Mar 15-8:47 AM

Given $\frac{1}{x} + \frac{1}{y} = 1$

Find $\frac{dy}{dx}$

$\frac{1}{x} + \frac{1}{y} = 1 \Rightarrow x^{-1} + y^{-1} = 1$

$\frac{d}{dx}\left[x^{-1} + y^{-1}\right] = \frac{d}{dx}[1]$

$\frac{d}{dx}[x^{-1}] + \frac{d}{dx}[y^{-1}] = 0$

$-1\,x^{-1-1} + (-1) \cdot y^{-1-1} \cdot \frac{dy}{dx} = 0$

$\frac{-1}{x^2} - \frac{1}{y^2} \cdot \frac{dy}{dx} = 0$

$\frac{-1}{y^2} \cdot \frac{dy}{dx} = \frac{1}{x^2}$ $\quad \frac{dy}{dx} = \frac{\frac{1}{x^2}}{\frac{-1}{y^2}}$

$\boxed{\frac{dy}{dx} = \frac{-y^2}{x^2}}$

Mar 15-8:49 AM

$y = \sin^2 x^3$

find $\frac{dy}{dx}$.

$y = [\cancel{\sin} x^3]^{\cancel{2}}$

$y' = 2[\sin x^3]^{2-1} \cdot \cos x^3 \cdot 3x^2$

$\boxed{y' = 6x^2 \cdot \sin x^3 \cdot \cos x^3}$

$y' = 3x^2 \cdot \underbrace{2 \sin x^3 \cos x^3}$

$\boxed{y' = 3x^2 \sin 2x^3}$

Mar 15-8:54 AM

Given $\cos y - \sqrt{x} = 5$

Find $\frac{dy}{dx}$

$\frac{d}{dx}[\cos y - \sqrt{x}] = \frac{d}{dx}[5]$

$\frac{d}{dx}[\cancel{\cos} \cancel{y}] - \frac{d}{dx}[x^{1/2}] = 0$

$-\sin y \cdot \frac{dy}{dx} \quad -\frac{1}{2}x^{-1/2} = 0$

$-\sin y \cdot \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}}}{-\sin y}$

$\boxed{\frac{dy}{dx} = \frac{-1}{2\sqrt{x}\sin y}}$

Mar 15-8:58 AM

Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[4]{x^2+8}}$

$$f(x) = (x^2+8)^{-1/4}$$

$$-\tfrac{1}{4} - 1 \to -\tfrac{5}{4}$$

$$\sqrt[4]{x^5} = \sqrt[4]{x^4}\,\sqrt[4]{x} = |x|\,\sqrt[4]{x}$$

$$f'(x) = \frac{-1}{\cancel{4}_{2}}(x^2+8)^{-1/4-1} \cdot (\cancel{2}x)$$

$$f'(x) = \frac{-x}{2}(x^2+8)^{-5/4} = \frac{-x}{2(x^2+8)^{5/4}}$$

$$= \frac{-x}{2\sqrt[4]{(x^2+8)^5}} = \frac{-x}{2|x^2+8|\sqrt[4]{x^2+8}}$$

$+$

$$= \boxed{\frac{-x}{2(x^2+8)\cdot\sqrt[4]{x^2+8}}}$$

Mar 15-9:02 AM

$\cos(xy) = 1 + \sin y$, find $\dfrac{dy}{dx}$

$$\frac{d}{dx}\left[\cancel{\cos}(xy)\right] = \frac{d}{dx}\left[1 + \sin y\right]$$

$$-\sin(xy)\cdot\left[1\cdot y + x\cdot\frac{dy}{dx}\right] = 0 + \cos y \cdot \frac{dy}{dx}$$

$$-y\sin(xy) - x\sin(xy)\cdot\frac{dy}{dx} = \cos y\cdot\frac{dy}{dx}$$

$$-y\sin(xy) = \cos y\cdot\frac{dy}{dx} + x\sin(xy)\cdot\frac{dy}{dx}$$

$$-y\sin(xy) = \left(\cos y + x\sin(xy)\right)\cdot\frac{dy}{dx}$$

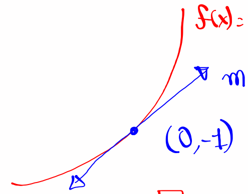
$$\boxed{\frac{-y\sin(xy)}{\cos y + x\sin(xy)} = \frac{dy}{dx}}$$

Mar 15-9:08 AM

Find eqn of the tan. line to the graph of $f(x) = \left(\dfrac{x-2}{x+2}\right)^5$ at $x = 0$.

1) $f(0) = \left(\dfrac{0-2}{0+2}\right)^5 = (-1)^5 = -1$ tan. Point $(0,-1)$

2) $f(x) = \left(\dfrac{x-2}{x+2}\right)^5$



$m = f'(0) = \dfrac{20(0-2)^4}{(0+2)^6}$

$= \dfrac{20(-2)^4}{2^6} = \dfrac{\cancel{20}^{5} \cdot \cancel{2^4}}{\cancel{2^6}_{4}}$

$\boxed{m = 5}$

$f(x) = \left(\boxed{\dfrac{x-2}{x+2}}\right)^{\boxed{5}}$

$f'(x) = 5\left(\dfrac{x-2}{x+2}\right)^4 \cdot \dfrac{\overbrace{1(x+2) - (x-2)\cdot 1}^{4}}{(x+2)^2}$

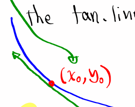
$f'(x) = \dfrac{5(x-2)^4 \cdot 4}{(x+2)^4 \cdot (x+2)^2}$ $\qquad f'(x) = \dfrac{20(x-2)^4}{(x+2)^6}$

$y - y_1 = m(x - x_1)$

$y - (-1) = 5(x - 0) \rightarrow \boxed{y = 5x - 1}$

Mar 15-9:14 AM

Find all points on the curve given by $x^2y^2 + xy = 2$ where the slope of the tan. line is -1.



$\dfrac{dy}{dx} = -1 \longrightarrow m = -1$

$\dfrac{d}{dx}\left[x^2y^2 + xy\right] = \dfrac{d}{dx}[2]$

$\dfrac{d}{dx}\left[x^2y^2\right] + \dfrac{d}{dx}[xy] = 0$

$2x \cdot y^2 + x^2 \cdot 2y \cdot \dfrac{dy}{dx} + 1 \cdot y + x \cdot \dfrac{dy}{dx} = 0$

$2xy^2 - 2x^2y + y - x = 0$

$2xy(y - x) + 1(y - x) = 0$

$(y - x)(2xy + 1) = 0$

by Zero-Product Rule

$y - x = 0$ OR $2xy + 1 = 0$

$\boxed{y = x}$ $\qquad y = \dfrac{-1}{2x}$

$x^2y^2 + xy = 2$

$x^2 \cdot x^2 + x \cdot x = 2$

$x^4 + x^2 - 2 = 0$

$(x^2 + 2)(x^2 - 1) = 0$

NO Real Soln. $\qquad x = \pm 1$

tan. Points where $m = -1$ → $(1,1)$, $(-1,-1)$

$x^2y^2 + xy = 2$

$\cancel{x^2} \cdot \dfrac{1}{4\cancel{x^2}} + \cancel{x} \cdot \dfrac{-1}{2\cancel{x}} = 2$

$\dfrac{1}{4} - \dfrac{1}{2} = 2$

false

Final Ans

$(1,1)$, $(-1,-1)$

Tan. lines

$y - 1 = -1(x - 1) \rightarrow \boxed{y = -x + 2}$

$y + 1 = -1(x + 1) \rightarrow \boxed{y = -x - 2}$

Mar 15-9:23 AM

Class QZ 6

Given $y = \sqrt{1+x^3}$

1) Find y when $x=2$.

$y = \sqrt{1+2^3} = \sqrt{9} = \boxed{3}$

2) Find $\frac{dy}{dx}$

$y = (1+x^3)^{1/2}$

$y' = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2\sqrt{1+x^3}}$

3) Find eqn of tan. line to the curve given by $y = \sqrt{1+x^3}$ at $x=2$.

$m = \frac{dy}{dx}\Big|_{x=2} = \frac{3 \cdot 2^2}{2\sqrt{1+2^3}} = \frac{3 \cdot 4}{2 \cdot 3} = \boxed{2}$

$y - 3 = 2(x-2) \longrightarrow \boxed{y = 2x - 1}$

Mar 15-9:42 AM